Valid Argument Forms

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An **argument** is a sequence of statements: one or more **premises**, followed by a **conclusion**. An argument is **valid** if whenever all the premises are true, the conclusion is also true — equivalently, if

$$(\text{Premise}_1 \wedge \text{Premise}_2 \wedge \cdots \wedge \text{Premise}_n) \to \text{Conclusion}$$

is a tautology.

Validity is about the argument's *form*, not the truth of its statements: an argument can be valid with a false premise or a false conclusion. It can also have true premises and a true conclusion and still be invalid, if the truth of the conclusion doesn't actually follow from the premises.

> **Provided in the exam.** The named forms below are on the *MATH1061/MATH7861 Examination Formula Page*, alongside the logical-equivalence-laws[1]. The rules of inference do **not** have to be memorised; the work left is recognising a form and naming it.

[1] courses/math1061/logical-equivalence-laws.pdf

How an argument is written down

The course lays an argument out with the premises stacked one per line and left-aligned, numbered `1. 2. 3.`, and the conclusion on the last line prefixed by $\therefore$ ("therefore") and left unnumbered. There is **no horizontal rule above the conclusion**:

$$\begin{aligned} &1.\quad p \to q \\ &2.\quad p \\ &\therefore\quad q \end{aligned}$$

The premise numbers are not decoration — they are what the justification column of a rules-of-inference proof refers back to ("from 1 and 4, by transitivity").

Why a false premise never threatens validity

In any row of the truth table where at least one premise is false, the conjunction of the premises is false — and F $\to X$ is true whatever X is. So that row of $(\text{Premise}_1 \wedge \cdots \wedge \text{Premise}_n) \to$ Conclusion is automatically T, no matter what the conclusion does there.

This is why the truth table method only inspects the rows where **all** premises are true: those are the only rows that can make the conditional false. It does *not* mean an argument with a false premise is automatically valid — a tautology needs every row true, and the all-premises-true rows still have to be checked.

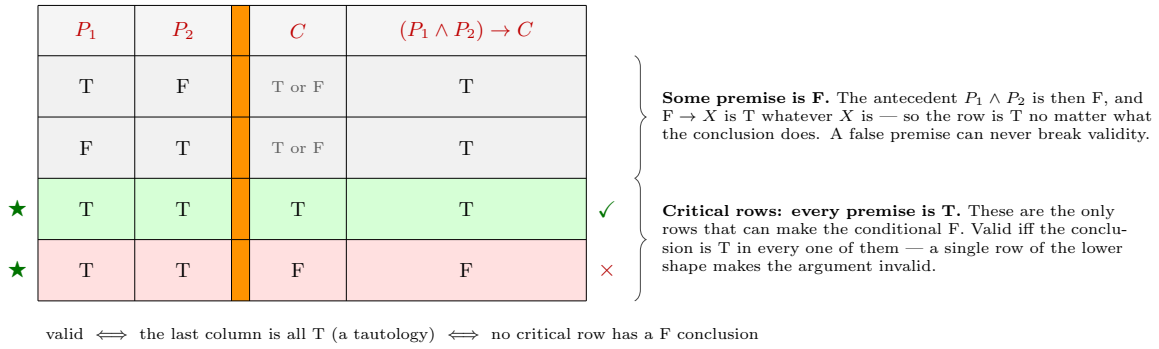

Figure 1: The definition of validity as a picture of a truth table. Rows where some premise is false (grey) make the antecedent of the packaged conditional false, so they are true for free and cannot break the tautology — which is why a false premise never threatens validity. Only the critical rows (starred, all premises T) decide the question. The amber band separates the premises from the conclusion. The red row is the one shape that would make an argument invalid.

Determining validity

- **Truth table method**: build a truth table for the premises and the conclusion. The argument is valid iff every row where all premises are T also has the conclusion T. Mark the all-premises-true rows with a in the margin and tick the conclusion in those rows, so the check is visible on the page.
- **Rules of inference method**: chain the laws of logical equivalence (see logical-equivalence-laws[2]) and the valid argument forms below to derive the conclusion from the premises. This proves validity; it can never prove *in*validity — failing to find a derivation is not evidence there isn't one.
- **Invalidity method**: assume all the premises are true and the conclusion is false, then reason out what that assumption forces each variable to be. Reaching a contradiction (some variable forced to be both T and F) means the assumption was impossible — the argument is **valid**. Reaching a consistent assignment instead gives a counterexample — the argument is **invalid**. This directed search is usually faster than building the full truth table once there are 3+ variables, and it is the only one of the three that hands you the counterexample when the argument fails.

Writing up a rules-of-inference proof

The layout is three columns: the **line number** down the left, the **derived statement** in the middle, and the **justification in its own right-hand column** — naming the law or form applied, and usually the lines it draws on (line 4 below is a step that names no line):

	statement	justification
1.	$w \to b$	premise 1
2.	$d \to\sim b$	premise 2
3.	$\sim(\sim b) \to\sim d$	from 2, by contrapositive
4.	$b \to\sim d$	by the double negative law

The last column is not optional: the **explanations must be written**. A chain of correct lines with no justifications is not a proof in this course.

Two rewrites that do most of the work

$$p \to q \ \equiv \sim p \vee q \ \equiv \sim q \to\sim p$$

Each has a name: $\sim p \vee q$ is the **definition** of $\to$, and $\sim q \to\sim p$ is the **contrapositive**. Between them they are what lets a conditional premise be turned into a disjunction (to

[2] courses/math1061/logical-equivalence-laws.pdf

reach a disjunctive conclusion) or flipped end-for-end (to line up with another conditional for Transitivity). Both are cited by name in the proof started above (in full in determining-argument-validity[3]) — the contrapositive at line 3, the definition of $\to$ at line 6 — though they are far from the only justifications a proof uses. See conditional-statements[4].

Valid argument forms

Name	Form(s)
Modus Ponens	$p \to q,\ p \therefore q$
Modus Tollens	$p \to q,\ \sim q \therefore \sim p$
Generalization	(a) $p \therefore p \vee q$; (b) $q \therefore p \vee q$
Specialization	(a) $p \wedge q \therefore p$; (b) $p \wedge q \therefore q$
Conjunction	$p,\ q \therefore p \wedge q$
Elimination	(a) $p \vee q,\ \sim q \therefore p$; (b) $p \vee q,\ \sim p \therefore q$
Transitivity	$p \to q,\ q \to r \therefore p \to r$
Proof by division into cases	$p \vee q,\ p \to r,\ q \to r \therefore r$
Contradiction Rule	$\sim p \to \mathbf{c} \therefore p$

These don't need to be memorised. Treat the table as a reference to scan rather than a checklist to recite: look down it for a form whose *conclusion* is the shape you want, then work out what the premises have to be turned into to fire it.

See also

- determining-argument-validity[5] — every Lecture 4 argument worked through, by all three methods
- logical-equivalence-laws[6]
- conditional-statements[7]
- 2026-08-03-valid-and-invalid-arguments[8] — Lecture 4
- practice-problems[9] — §4 Valid and Invalid Arguments, with full worked solutions

[3] courses/math1061/determining-argument-validity.pdf
[4] courses/math1061/conditional-statements.pdf
[5] courses/math1061/determining-argument-validity.pdf
[6] courses/math1061/logical-equivalence-laws.pdf
[7] courses/math1061/conditional-statements.pdf
[8] courses/math1061/2026-08-03-valid-and-invalid-arguments.pdf
[9] courses/math1061/practice-problems.pdf