Determining Argument Validity — Worked Examples

Table of contents

Worked example 1: the pink shirt argument 1
Worked example 2: a three-variable argument 3
Worked example 3: an argument via Conjunction, distribution, and Modus Ponens . 5
Worked example 4: an invalid argument 8
See also 10

Worked examples applying the methods from valid-argument-forms[1] — truth table, rules of inference, and the invalidity/counterexample search — on both valid and invalid arguments.

Worked example 1: the pink shirt argument

Consider the argument:

> If today is Monday, then I am wearing a pink shirt. Today is Monday. Therefore, I am wearing a pink shirt.

Let p = "today is Monday" and q = "I am wearing a pink shirt". In symbolic form:

1. $p \to q$
2. p
$\therefore$ q

This is the Modus Ponens form (see valid-argument-forms[2]).

[1] courses/math1061/valid-argument-forms.pdf
[2] courses/math1061/valid-argument-forms.pdf

Method 1: checking rows of the truth table

Build a truth table for the premises ($p \to q$ and p) and the conclusion (q). The same table serves all three columns — which column is a premise and which is the conclusion is something *you* impose on it, not something the table knows, so it is worth labelling the roles before checking anything:

p · *premise 2*	q · *conclusion*	$p \to q$ · *premise 1*
T	T	T
T	F	F
F	T	T
F	F	T

An argument is valid iff **every row where all the premises are true also has the conclusion true**. The premises here are $p \to q$ and p — both true only in row 1 (p=T, q=T, $p \to q$=T). In that row the conclusion q is also T, and there's no other row where both premises hold. So the argument is **valid**.

(To show an argument is *invalid* by this method, you instead go looking for a row where every premise is T but the conclusion is F. Worked examples 3 and 4 hunt for exactly that combination of truth values, but without building the table — see the invalidity method below.)

Method 2: checking the combined statement is a tautology

Equivalently — and this is the definition — the argument is valid exactly when the single statement

$$(\text{Premise}_1 \wedge \text{Premise}_2 \wedge \cdots \wedge \text{Last Premise}) \to \text{Conclusion}$$

is a **tautology**. For this argument that statement is $((p \to q) \wedge p) \to q$:

p	q	$p \to q$	$((p \to q) \wedge p) \to q$
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	T

Every row is T, so the statement is a **tautology** — the argument is valid. This is the same conclusion as Method 1, just packaged as one compound formula instead of a row-by-row check

across separate premise/conclusion columns. valid-argument-forms[3] has a figure showing why the two readings can never disagree: the rows Method 1 skips are precisely the rows Method 2 gets for free.

A valid argument can still have a false conclusion

This argument is valid regardless of whether p and q are actually true today — validity only says "*if* the premises hold, the conclusion follows", not that the premises actually do hold. The premise "if today is Monday, then I am wearing a pink shirt" might well be false in reality, and the argument built from it is still valid: the form guarantees the conclusion given the premises, not the truth of the premises themselves.

A false *conclusion* is fine because validity is a statement about the form. A false *premise* is fine because it makes the antecedent of (premises) $\rightarrow$ conclusion false in that row, and a conditional with a false antecedent is true — so that row is T whatever the conclusion does.

Worked example 2: a three-variable argument

Consider the argument:

> If wages are raised, buying increases. If there is a depression, buying does not increase. Therefore there is not a depression or wages are not raised.

Let w = "wages are raised", b = "buying increases", d = "there is a depression". (The letters are chosen as mnemonics of the English phrases — the course's habit whenever a problem starts in words.)

Translating to symbolic form

$$
\begin{array}{ll}
1. & w \rightarrow b \\
2. & d \rightarrow \sim b \\
\therefore & \sim d \vee \sim w
\end{array}
$$

[3] courses/math1061/valid-argument-forms.pdf

Truth table method

Three variables means $2^3 = 8$ rows in standard truth table form. The negations $\sim w$, $\sim b$, $\sim d$ are worth building as their own columns first — the premise and conclusion columns are then read straight off them rather than re-derived in your head. In the table below, the $\|$ column separates the input block (the variables and their negations) from the compound-form block (the two premises and the conclusion):

w	b	d	$\sim w$	$\sim b$	$\sim d$	$\|$	$w \to b$	$d \to\sim b$	$\sim d \vee \sim w$
T	T	T	F	F	F	$\|$	T	F	F
T	T	F	F	F	T	$\|$	T	T	T
T	F	T	F	T	F	$\|$	F	T	F
T	F	F	F	T	T	$\|$	F	T	T
F	T	T	T	F	F	$\|$	T	F	T
F	T	F	T	F	T	$\|$	T	T	T
F	F	T	T	T	F	$\|$	T	T	T
F	F	F	T	T	T	$\|$	T	T	T

Both premises ($w \to b$ and $d \to\sim b$) are true together in the four starred rows — 2, 6, 7, 8 — and the conclusion $\sim d \vee \sim w$ is also T in every one of them. There's no row where both premises hold and the conclusion fails, so the argument is **valid**. The other four rows are never inspected: each has a false premise, so it cannot break the argument (see valid-argument-forms[4]).

Note that this is *not* the same as the conclusion being always true — the conclusion is F in rows 1 and 3. Those rows simply have a false premise, so they say nothing about validity.

Rules of inference method

The premises are two conditionals, and the conclusion is a disjunction of two negations. Transitivity is the obvious form to aim at — but the two premises do not chain as they stand, and seeing why is the whole content of this problem:

Written out in the course's three-column layout:

	statement	justification
1.	$w \to b$	premise 1
2.	$d \to\sim b$	premise 2
3.	$\sim (\sim b) \to\sim d$	from 2, by contrapositive

[4] courses/math1061/valid-argument-forms.pdf

	statement	justification
4.	$b \rightarrow\sim d$	by the double negative law
5.	$w \rightarrow\sim d$	from 1 and 4, by Transitivity
6.	$\sim w \vee \sim d$	from 5, by the definition of $\rightarrow$
7.	$\sim d \vee \sim w$	from 6, by commutativity
	$\therefore\ \sim d \vee \sim w$	the argument is valid

Two things to copy from this write-up:

- **Write the contrapositive step out before simplifying it.** Line 3 is $\sim(\sim b) \rightarrow\sim d$ — the raw contrapositive — and only line 4 cancels the double negative. Collapsing the two makes the justification for line 5 harder to check, because Transitivity needs the forms $w \rightarrow b$ and $b \rightarrow\sim d$ to line up *exactly*.
- **Every line carries a justification.** The explanations must be written — the right-hand column is part of the answer, not commentary on it.

Invalidity method, as a third check

Suppose all the premises are true but the conclusion is false.

- Conclusion false $\Rightarrow \sim d \vee \sim w$ is false $\Rightarrow \sim d$ is false and $\sim w$ is false $\Rightarrow d$ is true and w is true.
- Premise 1 ($w \rightarrow b$) is true and w is true $\Rightarrow b$ is true.
- But now d is true and $\sim b$ is false, which makes premise 2 ($d \rightarrow\sim b$) false — contradicting the assumption that all the premises are true.

No consistent assignment exists, so the argument is **valid**. All three methods — truth table, rules of inference, invalidity search — agree.

Worked example 3: an argument via Conjunction, distribution, and Modus Ponens

Consider the argument (Q1 from the Methods video pre-class questions):

$$
\begin{array}{ll}
1. & p \rightarrow q \\
2. & p \vee r \\
3. & p \vee \sim r \\
\therefore & q
\end{array}
$$

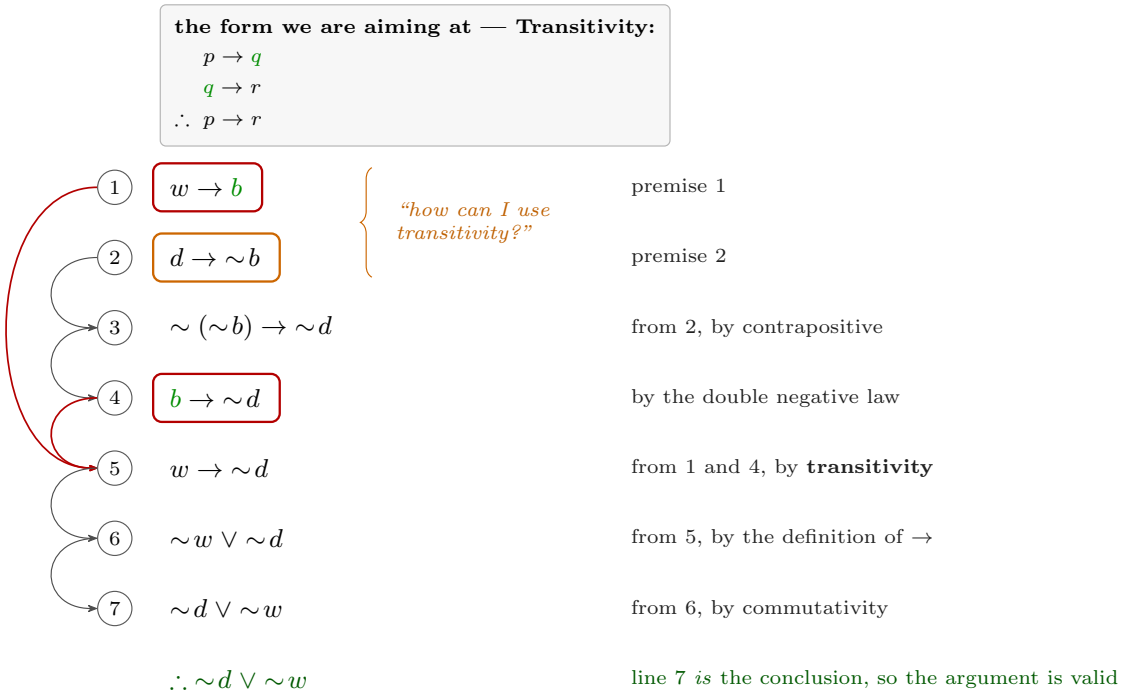


Figure 1: The proof in the course's three-column layout — line number, statement, justification — with the Transitivity form it aims at shown at the top. The red boxes mark lines 1 and 4, the two that Transitivity chains through the green b where they meet; the orange box marks line 2, the premise that has to be contraposed first. The arrows down the left margin show which earlier lines each step depends on.

Rules of inference method

The conclusion q is the consequent of premise 1, so Modus Ponens finishes the proof the moment p is available on its own. The whole question is how to manufacture p, and the answer is hiding in the two disjunctive premises.

	statement	justification
1.	$p \to q$	given premise
2.	$p \vee r$	given premise
3.	$p \vee \sim r$	given premise
4.	$(p \vee r) \wedge (p \vee \sim r)$	from 2 and 3, by Conjunction
5.	$p \vee (r \wedge \sim r)$	from 4, by the distributive law
6.	$p \vee \mathbf{c}$	from 5, by the negation law $(r \wedge \sim r \equiv \mathbf{c})$
7.	p	from 6, by the identity law
8.	q	from 1 and 7, by Modus Ponens

The trick is spotting that the two premises $p \vee r$ and $p \vee \sim r$ share the same left disjunct, so conjoining them and distributing p back out leaves "p, or a contradiction" — which forces p to be true regardless of r. The equivalence laws used at lines 5–7 are in logical-equivalence-laws[5].

Invalidity method, as a check

Suppose all the premises are true but the conclusion is false.

- Conclusion false $\Rightarrow$ q is false.
- Premise 1 $(p \to q)$ is true and q is false $\Rightarrow$ p is false.
- Premise 2 $(p \vee r)$ is true and p is false $\Rightarrow$ r is true.
- Premise 3 $(p \vee \sim r)$ is true and p is false $\Rightarrow$ $\sim r$ is true, i.e. r is false.

r can't be both true and false — impossible. No such truth values exist, so the argument is **valid**, agreeing with the rules-of-inference proof above.

[5] courses/math1061/logical-equivalence-laws.pdf

Worked example 4: an invalid argument

Not every argument is valid. Consider (Q2 from the Methods video pre-class questions):

1. $p \to q$
2. $q \to r$
3. $\sim p \vee \sim q$

$\therefore \quad r$

Invalidity method

Suppose all the premises are true but the conclusion is false.

- Conclusion false $\Rightarrow$ r is false.
- Premise 2 ($q \to r$) is true and r is false $\Rightarrow$ q is false.
- Premise 1 ($p \to q$) is true and q is false $\Rightarrow$ p is false.
- Premise 3 ($\sim p \vee \sim q$), checked against p false and q false: it's true. No contradiction anywhere.

This gives a **consistent** assignment — p, q, r all false — that makes every premise true and the conclusion false:

p	q	r	$p \to q$	$q \to r$	$\sim p \vee \sim q$	r
F	F	F	T	T	T	F

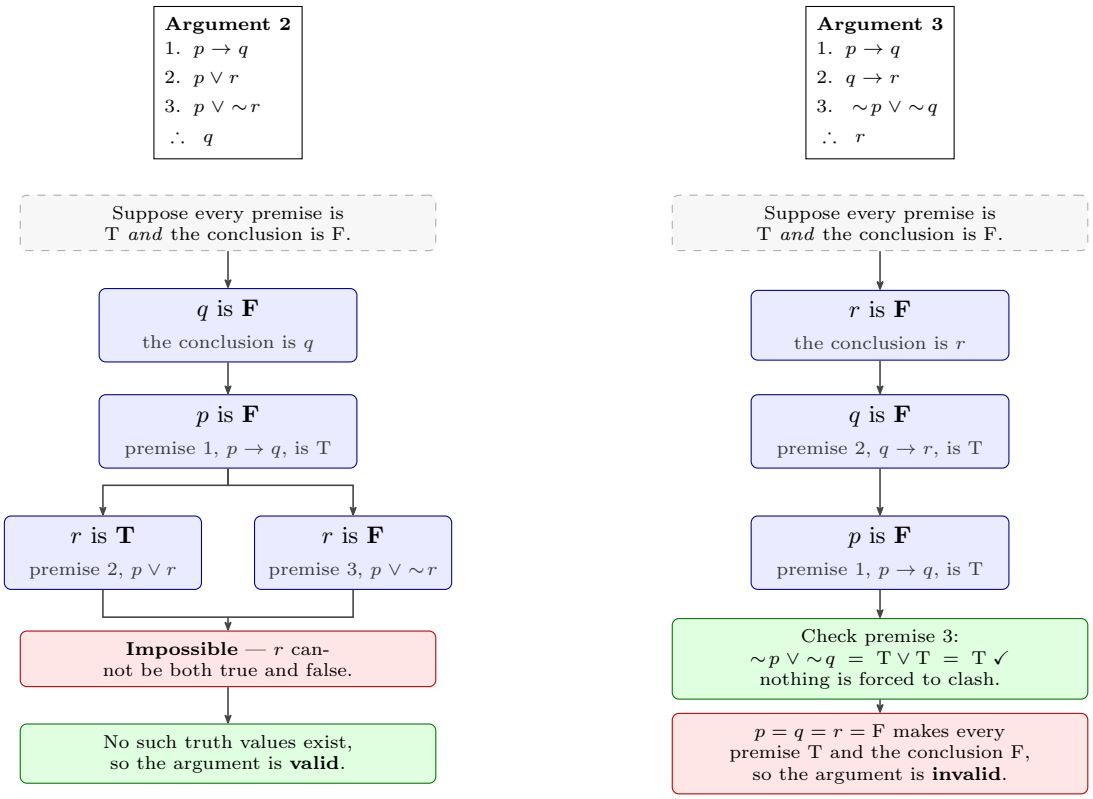
All three premises hold, the conclusion doesn't — that's a counterexample, so the argument is **invalid**.

Note the shape of the last step: once every variable has been forced, premise 3 has to be **checked**, not used to force anything further. If it had come out false the search would have closed with a contradiction instead, and the argument would have been valid.

How the search ends is the whole verdict

Worked examples 3 and 4 run the *same* procedure and part company only at the end. Setting them side by side is the clearest statement of what the method actually decides:

Because of that, the invalidity search is the method of choice when you don't yet know the answer: it settles the question either way, and when the argument fails it hands you the counterexample rather than just the news that no proof turned up. A rules-of-inference attempt that stalls tells you nothing.



Same procedure, opposite endings: the search for a counterexample either *closes* (some variable is forced both ways — there is no counterexample, so the argument is valid) or it *completes* (every variable gets a consistent value — that assignment *is* the counterexample, so the argument is invalid).

Figure 2: The invalidity search drawn as the cascade of forced values it produces, on the two arguments that end it differently. Assuming the conclusion false and every premise true forces variable after variable; the left-hand search (worked example 3) closes because r is forced both ways, so no counterexample exists and the argument is valid, while the right-hand search (worked example 4) runs to completion with every value consistent, and that assignment is the counterexample.

See also

- valid-argument-forms[6]
- logical-equivalence-laws[7]
- 2026-08-03-valid-and-invalid-arguments[8] — Lecture 4
- practice-problems[9] — §5 Methods for Determining Validity, with full worked solutions

[6] courses/math1061/valid-argument-forms.pdf
[7] courses/math1061/logical-equivalence-laws.pdf
[8] courses/math1061/2026-08-03-valid-and-invalid-arguments.pdf
[9] courses/math1061/practice-problems.pdf