Lecture 1 — Welcome & Warm-up Problems

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Two halves: administration — staff, expectations, support, assessment, the topic list and the schedule — and five warm-up problems that preview the rest of the semester. Everything administrative is in math1061[1]. The few course rules that matter most are repeated here, followed by the problems.

Objectives for the hour

- Introduce the lecturing staff.
- Review course expectations, the course profile and the course website.
- Support mechanisms.
- Explain the assessment structure.
- Topics overview.

The load-bearing bits of the course rules:

- **Lectures are Mon 10am, Tue 2pm, Thu 1pm** — see the table in math1061[2].
- **General enquiries go to a course address, not a personal one**: `math1061@uq.edu.au` and `math7861@uq.edu.au`.

[1] courses/math1061/math1061.pdf
[2] courses/math1061/math1061.pdf

- **The pre-class hour is two things**: watch videos **and** answer multiple-choice questions. Both, not just the videos.
- Among the activities: the **Active Class**, and **Complete Assessed Exercises**.
- In the assessment: **best 8 of 12**, and the **hurdles** on the final exam.

Warm-up problems

1. Let p, q and r be consecutive positive integers with $p < q < r$, and let

$$a = p + r, \qquad b = pr, \qquad c = r - p.$$

   Determine which of a, b, c must be even.

2. Four non-zero real numbers a, b, c, d are given, where $b + d \neq 0$. If $\dfrac{a}{b} + \dfrac{c}{d} = \dfrac{a+c}{b+d}$, show that $ac < 0$.

3. True or False: In every group of five people, there are two people who have the same number of friends within the group.

4. **The Tower of Hanoi**: Given a tower of 8 discs in decreasing size on one of three pegs, transfer the entire tower to one of the other pegs according to the following rules: (a) Move only one disc at a time. (b) Never move a larger disc onto a smaller disc. Is there a solution? If there is, what is the fewest number of moves needed to complete the transfer?

5. **The Königsberg Bridge Problem**: Does there exist a route through the city that crosses each bridge exactly once and takes you back to the starting place? (No turning around on bridges.)

The short answers to the last three: **3** — **True**, **4** — **255**, **5** — **No**. Problems 1 and 2 are worked in full below.

Solution 1 — parity of $p + r$, pr and $r - p$

Start with examples, not algebra: $1, 2, 3$, then $5, 6, 7$, then $2, 3, 4$. In each triple look at p **and** r, the first and last. What matters is their **parity** (even/odd): p and r are always two apart, so they are always the same kind of number.

Written out: p, q, r are consecutive, so

$$q = p + 1 \qquad \text{and} \qquad r = p + 2.$$

$a = p + r$ **is always even.** Since $r = p + 2$, the numbers p and r are both even or both odd, and even + even = even and odd + odd = even. Either way a is even.

$b = pr$ **is either even or odd**, and that is the whole answer. If p and r are even then pr is even; if both are odd then pr is odd. Nothing forces one case over the other, so b need not be even.

$c = r - p$ **is always even**, and more than that, it is always exactly 2:

$$c = r - p = (p + 2) - p = 2.$$

So a **and** c **must be even;** b **need not be.**

consecutive $\Rightarrow q = p + 1$ and $r = p + 2$, so p and r are two apart
$\Rightarrow p, r$ always share their parity, and $c = r - p = 2$ always

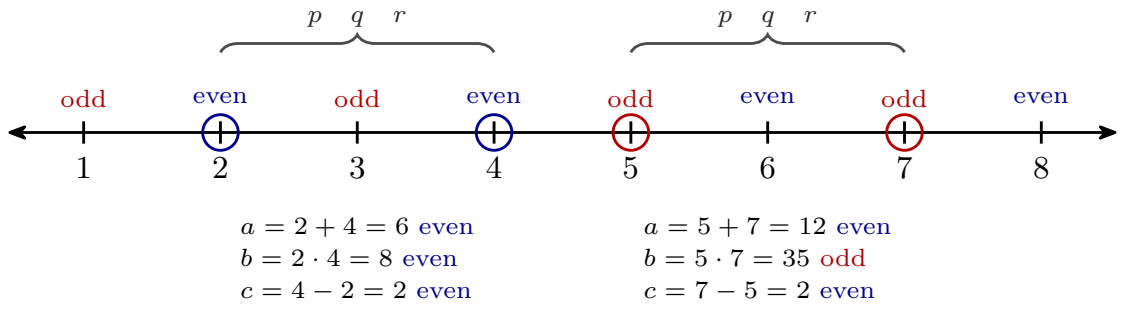


Figure 1: Why two of the three expressions are forced. Consecutive integers alternate parity, so p and r — two apart — always match, which makes p+r even and pinches r−p down to the constant 2; only the product pr is free to go either way. The two windows are the triples 2,3,4 and 5,6,7, with p and r ringed and a, b, c worked out beneath each. Blue marks even and red odd throughout.

Nothing in this solution is more than the definition of "consecutive" plus the two parity facts. That is the shape every direct proof in proof-techniques[3] takes from Lecture 7 onwards. For b, *"either even or odd"* is a complete answer to "which **must** be even", because a single pair of examples on opposite sides settles it.

Solution 2 — showing $ac < 0$

The proof is one algebraic chain, ending in an argument about signs:

Suppose $\dfrac{a}{b} + \dfrac{c}{d} = \dfrac{a + c}{b + d}$.

[3] courses/math1061/proof-techniques.pdf

Put the left side over a common denominator:

$$\frac{ad}{bd} + \frac{bc}{bd} = \frac{a+c}{b+d}$$

Multiply both sides by $bd(b+d)$ — legitimate because $b, d \neq 0$ and $b+d \neq 0$, which is exactly why the question hands you those conditions. The bd cancels on the left and the $b+d$ on the right:

$$bd\,(b+d)\left(\frac{ad+bc}{bd}\right) = \left(\frac{a+c}{b+d}\right)(b+d)\,bd$$

$$(ad+bc)(b+d) = (a+c)\,bd$$

Expand the left side (**FOIL**):

$$abd + ad^2 + b^2c + bcd = abd + bcd$$

The abd and bcd terms cancel from both sides, leaving

$$ad^2 + b^2c = 0, \qquad \text{i.e.} \qquad ad^2 = -b^2c.$$

Divide both sides by ab^2 (allowed: $a \neq 0$ and $b \neq 0$):

$$\frac{d^2}{b^2} = -\frac{c}{a}$$

The left side is a square over a square with $d \neq 0$, so it is **positive**; therefore $-\frac{c}{a}$ is positive, so $\frac{c}{a}$ is **negative**. A negative quotient means **exactly one of a and c is negative**, so their product is negative. $\therefore\ ac < 0$.

Two habits here that the rest of the course keeps asking for. First, every non-zero hypothesis gets used at the moment it licenses a division — the three conditions $b \neq 0$, $d \neq 0$, $b+d \neq 0$ are not decoration. Second, the final step reasons about **sign**, not about values: $\frac{c}{a} < 0$ says one of the two is negative without saying which, and that is already enough for $ac < 0$.

The symbol $\therefore$ reads "Therefore". It prefixes the conclusion of an argument all semester (see math1061-diagram-notation[4]).

Solutions 3, 4 and 5

Each of these answers is the punchline of a topic later in the semester.

[4] courses/math1061/math1061-diagram-notation.pdf

3. True — and it is the pigeonhole principle

True. Each of the five people has a friend count somewhere in $\{0, 1, 2, 3, 4\}$ — five people, five possible values, so counting alone is not enough. The extra fact is that 0 **and** 4 **cannot both occur**: if someone is friends with all four others, then nobody has zero friends; and if someone has zero friends, nobody can be friends with all four. So the counts actually available are only four values, and five people distributed over four values must repeat one. That is the pigeonhole principle, which arrives in **Lecture 33** — see pigeonhole-principle[5], where the same argument is made for the degrees of a graph, and practice-problems[6].

(This assumes friendship is mutual, which is what "friends within the group" is taken to mean.)

4. 255 moves

255. The reasoning comes in Lecture 16 and is worked in full at solving-recurrence-relations[7]: letting m_n be the minimum number of moves for n discs, moving the largest disc forces the other $n-1$ onto the spare peg and back again, giving

$$m_1 = 1, \qquad m_k = 2m_{k-1} + 1 \ \text{ for } k \geq 2,$$

whose explicit formula is $m_n = 2^n - 1$. For eight discs, $2^8 - 1 = 255$. The answer to "is there a solution?" is yes, and constructively so — the recursion *is* the algorithm.

5. No

No. The modelling step is to redraw the map of Königsberg, a river crossed by seven bridges, as a graph. Graph theory is introduced at Lecture 33; the graph theory lectures are L33–L36. Graphs, loops, multiple edges and degree are defined in graphs[8].

The question as asked is for a **closed** route — one that returns to where it started. Such a route must leave every land mass exactly as often as it enters it, so every bridge at a land mass is used in an entry/exit pair, and **every vertex must have even degree**. Here all four are odd, so no closed route exists.

Keep the two criteria apart. The standard slogan is about the *open* case: an open route crossing every bridge once (not required to return) exists only when the number of odd-degree vertices is 0 or 2. Königsberg has four, so that fails too — but "more than two odd vertices" is not the criterion for the question as asked.

[5] courses/math1061/pigeonhole-principle.pdf
[6] courses/math1061/practice-problems.pdf
[7] courses/math1061/solving-recurrence-relations.pdf
[8] courses/math1061/graphs.pdf

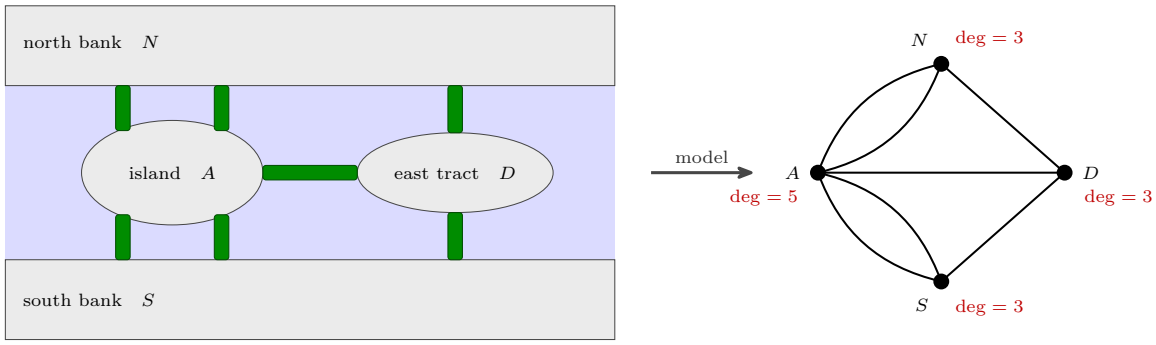


Figure 2: The modelling step. Each land mass becomes a vertex and each bridge an edge, so the seven bridges become seven edges on four vertices — and the question 'can I walk every bridge once and get back?' becomes a question about degrees. Every vertex here has odd degree, and a closed route must enter and leave each land mass in pairs, so no such route exists. The left panel is a schematic of the city, with the land masses named A, N, S and D and the bridges in green; the right panel is the same city as a graph.

Both criteria are the two halves of one theorem about connected graphs: an **Euler circuit** exists exactly when every vertex has even degree, and an **Euler trail** between two different vertices exactly when those two are the only odd ones. The theorem, its proof and how to find such a route are in euler-circuits-and-trails[9], and Lecture 34 (2026-10-26-walks-trails-and-circuits[10]) applies it.

What each problem previews

Problem	Previews	Lecture
1 — parity of $p+r$, pr, $r-p$	direct proof, definitions of even/odd, number theory	L7 (proof-techniques[11])
2 — showing $ac < 0$	direct proof from an algebraic hypothesis; using every hypothesis	L7 (proof-techniques[12])

[9] courses/math1061/euler-circuits-and-trails.pdf
[10] courses/math1061/2026-10-26-walks-trails-and-circuits.pdf
[11] courses/math1061/proof-techniques.pdf
[12] courses/math1061/proof-techniques.pdf

Problem	Previews	Lecture
3 — five people, same friend count	the pigeonhole principle	L33 (pigeonhole-principle[13])
4 — Tower of Hanoi	recursion, recurrence relations, induction	L13–L16 (solving-recurrence-relations[14])
5 — Königsberg bridges	graph theory, walks and circuits	L33–L34 (graphs[15], euler-circuits-and-trails[16])

That table is the course outline in miniature — logic and proof first, then number theory, sets, functions and relations, group theory, counting, and graph theory last. Fields and probability belong to the same stretch of the semester; see math1061[17] for the course profile's topic list. Some of this **overlaps Year 11 and 12 Specialist Mathematics**.

See also

- math1061[18] — staff, assessment weights, support, and the lecture schedule
- math1061-diagram-notation[19] — the course's own notation and figure conventions
- 2026-07-30-logical-form-and-equivalence[20] — Lecture 2, where the symbolic logic starts properly
- proof-techniques[21] — the four proof methods problems 1 and 2 are a warm-up for
- solving-recurrence-relations[22] — the Tower of Hanoi recurrence worked in full
- euler-circuits-and-trails[23] — the theorem that settles the bridges of Königsberg
- practice-problems[24] — the semester-wide practice problem set, with solutions; it belongs to the individual-study hours, not the class (the logic sections begin at practice-problems[25])

[13] courses/math1061/pigeonhole-principle.pdf
[14] courses/math1061/solving-recurrence-relations.pdf
[15] courses/math1061/graphs.pdf
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